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Fourier Series Expansion with Adjustable Bandwidth: An Innovative Algorithm

Cheng-Che Lee, Hsin-Jung Lee*, Yi-Min Yang, Wei-Yu Lee, Chieh-Hsiung Kuan

Graduate Institute of Electrics Engineering, National Taiwan University, Taipei 106, Taiwan (R.O.C.)
*** E-Mail: d04943010@ntu.edu.tw**

Abstract

We introduced a novel correction term basis that improves upon the traditional Fourier series expansion, achieving completeness in the expansion with an infinite number of terms. The root-mean-square error between the target function and the simulated function can reach below 10^{-5}, which facilitates improved signal analysis for wearable electrocardiogram (ECG) measurement systems.

Fourier Series and Correction Term Basis

A new set of bases, called **Correction Term $g(x)$**, was introduced into the original Fourier series expansion terms, written as follow.

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{N_1}\left(a_n cos\frac{2\pi nx}{T} + b_n sin\frac{2\pi nx}{T}\right) + g(x) \qquad (1)$$

F(x) is the objective function to be expanded, N_1 is the number of terms in Fourier expansion, which is a positive integer, and T is the Fourier expansion period.

The correction term $g(x)$, was listed in the equation (2)

$$g(x) = A_1(cos2\pi f_{c_1}x) + B_1(sin2\pi f_{c_1}x) + A_2(cos2\pi f_{c_2}x) + B_2(sin2\pi f_{c_2}x) + \cdots + A_{N_2}\left(cos2\pi f_{cN_2}x\right) + B_{N_2}\left(sin2\pi f_{cN_2}x\right) \qquad (2)$$

Unlike the frequency of Fourier series expansion, which is an integer multiple of the reciprocal of the expansion period, the frequency used in **the correction frequency f_c** is the frequency between the reciprocals of each expansion period.

A_1, B_1, A_2, B_2...A_{N2}, B_{N2} are the coefficients of the expansion base, f_{c1}, f_{c2}...f_{cN2} are the frequencies of expansion, and N_2 is the number of expansion terms.

Selection of Frequency

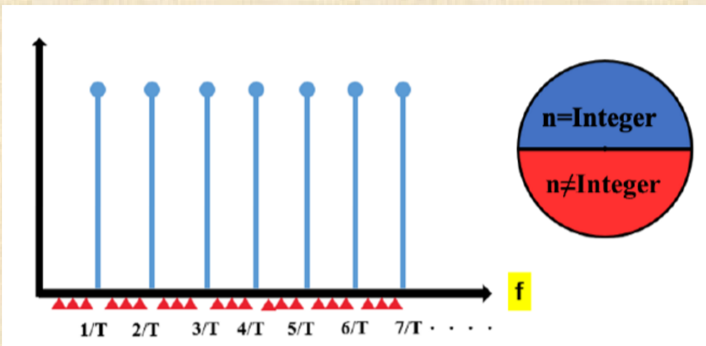
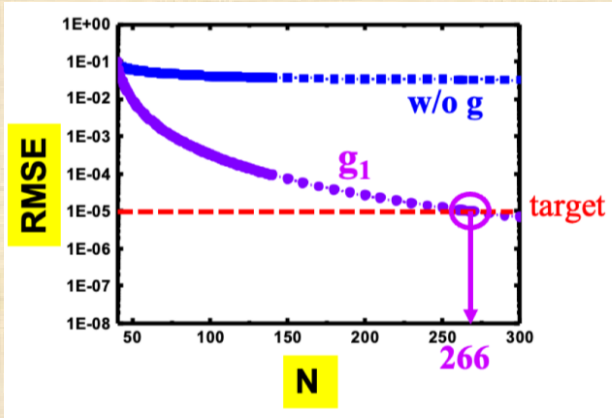


Fig. 1 The schematic of the spectrum with the introduction of the correction term basis.

In the case of using these two bases, the objective function can be completely expressed, and the completeness of limited bandwidth can be established.

Results

The relationship between the number of terms in the Fourier series expansion and the root-mean-square error (RMSE) was investigated after adding a set of correction terms based on Equation (1).



- $F(x) = \sqrt{0.5} \times cos(2\pi f x) + \sqrt{0.5} \times sin(2\pi f x$
- the f of the target function = 29.5 Hz.
- The Fourier expansion period T = 1 second
- expansion over the range x = - 0.5 to 0.5.
- cutoff frequency = 40 Hz.

Fig. 2 The number of terms in the Fourier series expansion and the root mean square error for a set of correction terms.

We incorporated correction terms and discussed the scenario involving a set of correction terms. A correction frequency $fc1$ of 39.5 Hz was chosen here, located in the vicinity of 39 Hz and 40 Hz, as the correction frequency.

From Figure 2, it can be observed that when the original Fourier series is expanded to 300 terms, the error begins to converge between 10^{-1} and 10^{-2}. After adding a set of correction terms, achieving our target RMSE of 10^{-5} can be accomplished with an expansion of around 266 terms.

Application to ECG signals analysis

An ECG signal from a different subject was randomly selected, and the R-wave from the QRS complex extracted.

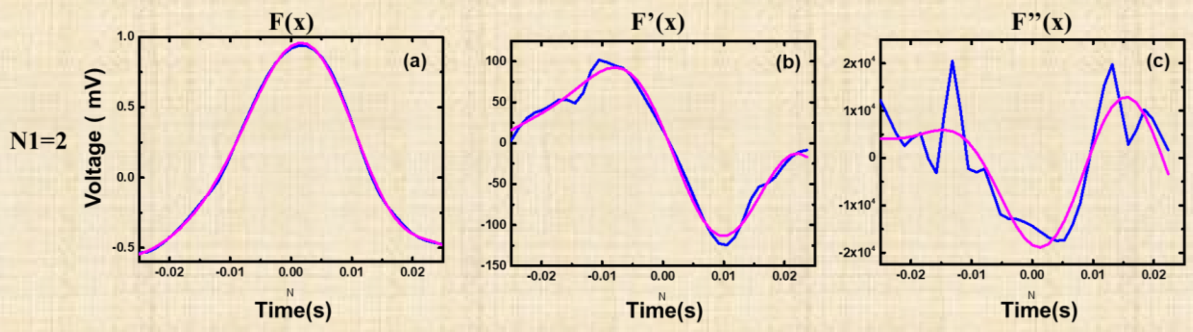


Fig. 3 (a) An original ECG signal sourced from the MIT-BIH Arrhythmia Database, (b) the first order and (c) the second order derivative of the ECG signal. (T=0.05 s, 20 data points.)

Matrix methods were used to calculate all the coefficients for the Fourier series expansion and the correction terms. It can be observed that when the Fourier series expansion is limited to a frequency bandwidth of 40 Hz, the differential curves generated using our algorithm **appear smoother** compared to the original data's differential curves.

Conclusion

We introduced a novel correction term basis that improves upon the traditional Fourier series expansion, achieving completeness in the expansion with an infinite number of terms and applied this algorithm to analyze ECG signals.

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